

**Yusra Djabri, Fazia Bedouhene, Fatiha Boulahia**  
*Further properties of Stepanov–Orlicz almost periodic functions*

Comment.Math.Univ.Carolin. 61,3 (2020) 363–382.

**Abstract:** We revisit the concept of Stepanov–Orlicz almost periodic functions introduced by Hillmann in terms of Bochner transform. Some structural properties of these functions are investigated. A particular attention is paid to the Nemytskii operator between spaces of Stepanov–Orlicz almost periodic functions. Finally, we establish an existence and uniqueness result of Bohr almost periodic mild solution to a class of semilinear evolution equations with Stepanov–Orlicz almost periodic forcing term.

**Keywords:** Bohr almost periodic; Bochner transform; Stepanov–Orlicz almost periodic function; semilinear evolution equations; Nemytskii operator

**AMS Subject Classification:** 34C27, 35B15, 46E30

REFERENCES

- [1] Albrycht J., *The theory of Marcinkiewicz–Orlicz spaces*, Rozprawy Mat. **27** (1962), 56 pages.
- [2] Amerio L., Prouse G., *Almost-Periodic Functions and Functional Equations*, Van Nostrand Reinhold, New York, Ont.-Melbourne, 1971.
- [3] Andres J., Bersani A. M., Grande R. F., *Hierarchy of almost-periodic function spaces*, Rend. Mat. Appl. (7) **26** (2006), no. 2, 121–188.
- [4] Andres J., Pennequin D., *On Stepanov almost-periodic oscillations and their discretizations*, J. Difference Equ. Appl. **18** (2012), no. 10, 1665–1682.
- [5] Andres J., Pennequin D., *On the nonexistence of purely Stepanov almost-periodic solutions of ordinary differential equations*, Proc. Amer. Math. Soc. **140** (2012), no. 8, 2825–2834.
- [6] Bedouhene F., Challali N., Mellah O., Raynaud de Fitte P., Smaali M., *Almost periodic solution in distribution for stochastic differential equations with Stepanov almost periodic coefficients*, available at arXiv: 1703.00282v3 [math.PR] (2017), 42 pages.
- [7] Bugajewski D., Nawrocki A., *Some remarks on almost periodic functions in view of the Lebesgue measure with applications to linear differential equations*, Ann. Acad. Sci. Fenn., Math. **42** (2017), no. 2, 809–836.
- [8] Chen S., *Geometry of Orlicz Spaces*, Dissertationes Math. (Rozprawy Mat.), 356, 1996.
- [9] Cichoń M., Metwali M. M. A., *On quadratic integral equations in Orlicz spaces*, J. Math. Anal. Appl. **387** (2012), no. 1, 419–432.
- [10] Corduneanu C., *Almost Periodic Functions*, Interscience Tracts in Pure and Applied Mathematics, 22, Interscience Publishers, John Wiley, New York, 1968.
- [11] Dads A. E. H., Es-Sebbar B., Ezzinbi K., Ziat M., *Behavior of bounded solutions for some almost periodic neutral partial functional differential equations*, Math. Methods Appl. Sci. **40** (2017), no. 7, 2377–2397.
- [12] Danilov L. I., *On the uniform approximation of a function that is almost periodic in the sense of Stepanov*, Izv. Vyssh. Uchebn. Zaved. Mat (1998), no. 5, 10–18.
- [13] Diagana T., *Stepanov-like pseudo-almost periodicity and its applications to some nonautonomous differential equations*, Nonlinear Anal. **69** (2008), no. 12, 4277–4285.
- [14] Diagana T., Zitane M., *Stepanov-like pseudo-almost automorphic functions in Lebesgue spaces with variable exponents  $L^{p(x)}$* , Electron. J. Differential Equations **2013** (2013), No. 188, 20 pages.
- [15] Ding H.-S., Long W., N’Guérékata G. M., *Almost periodic solutions to abstract semilinear evolution equations with Stepanov almost periodic coefficients*, J. Comput. Anal. Appl. **13** (2011), no. 2, 231–242.
- [16] Hillmann T. R., *Besicovitch–Orlicz spaces of almost periodic functions*, Real and stochastic analysis, Wiley Ser. Probab. Math. Statist. Probab. Math. Statist., Wiley, 1986, 119–167.
- [17] Hu Z., *Boundedness and Stepanov’s almost periodicity of solutions*, Electron. J. Differential Equations **2005** (2005), no. 35, 7 pages.
- [18] Hu Z., Mingarelli A. B., *Bochner’s theorem and Stepanov almost periodic functions*, Ann. Mat. Pura Appl. (4) **187** (2008), no. 4, 719–736.

- [19] Hudzik H., *Uniform convexity of Musielak–Orlicz spaces with Luxemburg’s norm*, Comment. Math. Prace Mat. **23** (1983), no. 1, 21–32.
- [20] Kasprzak P., Nawrocki A., Signerska-Rynkowska J., *Integrate-and-fire models with an almost periodic input function*, J. Differential Equations **264** (2018), no. 4, 2495–2537.
- [21] Kourat H., *Caractérisation de quelques propriétés géométriques locales dans les espaces de type Musielak–Orlicz*, PhD. Thesis, Mouloud Mammeri University of Tizi–Ouzou, Tizi–Ouzou, 2016 (French).
- [22] Kozłowski W. M., *Modular Function Spaces*, Monographs and Textbooks in Pure and Applied Mathematics, 122, Marcel Dekker, New York, 1988.
- [23] Kufner A., John O., Fučík S., *Function Spaces*, Monographs and Textbooks on Mechanics of Solids and Fluids, Mechanics: Analysis, Noordhoff International Publishing, Leyden, Publishing House of the Czechoslovak Academy of Sciences, Prague, 1977.
- [24] Levitan B. M., Zhikov V. V., *Almost Periodic Functions and Differential Equations*, Cambridge University Press, Cambridge, 1982.
- [25] Luxemburg W. A. J., *Banach Function Spaces*, PhD. Dissertation, Delft University of Technology, Delft, 1955.
- [26] Musielak J., *Orlicz Spaces and Modular Spaces*, Lecture Notes in Mathematics, 1034, Springer, Berlin, 1983.
- [27] Pankov A. A., *Bounded and Almost Periodic Solutions of Nonlinear Operator Differential Equations*, Mathematics and Its Applications (Soviet Series), 55, Kluwer Academic Publishers Group, Dordrecht, 1990.
- [28] Radová L., *Theorems of Bohr–Neugebauer-type for almost-periodic differential equations*, Math. Slovaca **54** (2004), no. 2, 191–207.
- [29] Rao A. S., *On the Stepanov-almost periodic solution of a second-order operator differential equation*, Proc. Edinburgh Math. Soc. (2) **19** (1974/75), 261–263.
- [30] Stepanoff W., *Über einige Verallgemeinerungen der fast periodischen Funktionen*, Math. Ann. **95** (1926), no. 1, 473–498 (German).
- [31] Stoiński S., *Almost periodic functions in the Lebesgue measure*, Comment. Math. (Prace Mat.) **34** (1994), 189–198.
- [32] Zaidman S., *An existence result for Stepanoff almost-periodic differential equations*, Canad. Math. Bull. **14** (1971), 551–554.