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Fixed point approximation under Mann iteration beyond Ishikawa

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Abstract: Consider the Mann iteration $x_{n+1} = (1 - \alpha_n)x_n + \alpha_n T x_n$ for a nonexpansive mapping $T: K \rightarrow K$ defined on some subset K of the normed space X . We present an innovative proof of the Ishikawa almost fixed point principle for nonexpansive mapping that reveals deeper aspects of the behavior of the process. This fact allows us, among other results, to derive convergence of the process under the assumption of existence of an accumulation point of $\{x_n\}$.

Keywords: Mann iteration; fixed point; nonexpansive mapping

AMS Subject Classification: 47H10

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