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Approximating solutions of split equality of some nonlinear optimization problems using an inertial algorithm

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Abstract: This paper presents an inertial iterative algorithm for approximating a common solution of split equalities of generalized mixed equilibrium problem, monotone variational inclusion problem, variational inequality problem and common fixed point problem in real Hilbert spaces. The algorithm is designed in such a way that it does not require prior knowledge of the norms of the bounded linear operators. We prove a strong convergence theorem under some mild conditions of the control sequences and also give a numerical example to show the efficiency and accuracy of our algorithm. We see that the inertial algorithm performs better in terms of number of iteration and CPU-time than the non-inertial algorithm. This result improves and generalizes many recent results in the literature.

Keywords: split equality; generalized equilibrium problem; variational inclusion problem; variational inequality; quasi-nonexpansive mapping; fixed point problem
AMS Subject Classification: 47H06, 47H09, 47J05, 47J25

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