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Nonassociative triples in involutory loops and in loops of small order

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Abstract: A loop of order n possesses at least $3n^2-3n+1$ associative triples. However, no loop of order $n>1$ that achieves this bound seems to be known. If the loop is involutory, then it possesses at least $3n^2-2n$ associative triples. Involutory loops with $3n^2-2n$ associative triples can be obtained by prolongation of certain maximally nonassociative quasigroups whenever $n-1$ is a prime greater than or equal to 13 or $n-1=p^{2k}$, p an odd prime. For orders $n \leq 9$ the minimum number of associative triples is reported for both general and involutory loops, and the structure of the corresponding loops is described.

Keywords: quasigroup; loop; prolongation; involutory loop; associative triple; maximally nonassociative

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