

Aleš Drápal, Michael Kinyon
Normality, nuclear squares and Osborn identities

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Abstract: Let Q be a loop. If $S \leq Q$ is such that $\varphi(S) \subseteq S$ for each standard generator of $\text{Inn } Q$, then S does not have to be a normal subloop. In an LC loop the left and middle nucleus coincide and form a normal subloop. The identities of Osborn loops are obtained by applying the idea of nuclear identification, and various connections of Osborn loops to Moufang and CC loops are discussed. Every Osborn loop possesses a normal nucleus, and this nucleus coincides with the left, the right and the middle nucleus. Loops that are both Buchsteiner and Osborn are characterized as loops in which each square is in the nucleus.

Keywords: loop; normal subloop; LC loop; Buchsteiner loop; Osborn loop; nuclear identification

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