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Inverse property of nonassociative abelian extensions

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Abstract: Our paper deals with the investigation of extensions of commutative groups by loops so that the quasigroups that result in the multiplication between cosets of the kernel subgroup are T-quasigroups. We limit our study to extensions in which the quasigroups determining the multiplication are linear functions without constant term, called linear abelian extensions. We characterize constructively such extensions with left-, right-, or inverse properties using a general construction according to an equivariant group action principle. We show that the obtained constructions can be simplified for ordered loops. Finally, we apply our characterization to determine the possible cardinalities of the component loop of finite linear abelian extensions.

Keywords: loop; nonassociative extensions of abelian groups; linear abelian extensions; left property; right property; inverse property
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