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Automorphic loops and metabelian groups

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Abstract: Given a uniquely 2-divisible group G, we study a commutative loop $(G, \circ)$ which arises as a result of a construction in "Engelsche elemente noetherscher gruppen" (1957) by R. Baer. We investigate some general properties and applications of "$\circ$" and determine a necessary and sufficient condition on G in order for $(G, \circ)$ to be Moufang. In "A class of loops categorically isomorphic to Bruck loops of odd order" (2014) by M. Greer, it is conjectured that G is metabelian if and only if $(G, \circ)$ is an automorphic loop. We answer a portion of this conjecture in the affirmative: in particular, we show that if G is a split metabelian group of odd order, then $(G, \circ)$ is automorphic.

Keywords: metabelian groups; automorphic loops; Bruck loops; Moufang loops
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