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*Semisymmetrization and Mendelsohn quasigroups*

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**Abstract:** The semisymmetrization of an arbitrary quasigroup builds a semisymmetric quasigroup structure on the cube of the underlying set of the quasigroup. It serves to reduce homotopies to homomorphisms. An alternative semisymmetrization on the square of the underlying set was recently introduced by A. Krapež and Z. Petrić. Their construction in fact yields a Mendelsohn quasigroup, which is idempotent as well as semisymmetric. We describe it as the Mendelsohnization of the original quasigroup. For quasigroups isotopic to an abelian group, the relation between the semisymmetrization and the Mendelsohnization is studied. It is shown that the semisymmetrization is the total space for an action of the Mendelsohnization on the abelian group. The Mendelsohnization of an abelian group isotope is then identified as the idempotent replica of its semisymmetrization, with fibers isomorphic to the abelian group.

**Keywords:** semisymmetric; quasigroup; Mendelsohn triple system
**AMS Subject Classification:** 20N05

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