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Remarks on WDC sets

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Abstract: We study WDC sets, which form a substantial generalization of sets with positive reach and still admit the definition of curvature measures. Main results concern WDC sets $A \subset \mathbb{R}^2$. We prove that, for such A, the distance function $d_A = \operatorname{dist}(\cdot, A)$ is a "DC aura" for A, which implies that each closed locally WDC set in $\mathbb{R}^2$ is a WDC set. Another consequence is that compact WDC subsets of $\mathbb{R}^2$ form a Borel subset of the space of all compact sets.

Keywords: distance function; WDC set; DC function; DC aura; Borel complexity
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