José G. Anaya, Enrique Castañeda-Alvarado, José A. Martínez-Cortez

On the hyperspace $C_n(X)/C_{nK}(X)$

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Abstract: Let X be a continuum and n a positive integer. Let $C_n(X)$ be the hyperspace of all nonempty closed subsets of X with at most n components, endowed with the Hausdorff metric. For K compact subset of X, define the hyperspace $C_{nK}(X) = \{A \in C_n(X) \colon K \subset A\}$. In this paper, we consider the hyperspace $C_K^n(X) = C_n(X)/C_{nK}(X)$, which can be a tool to study the space $C_n(X)$. We study this hyperspace in the class of finite graphs and in general, we prove some properties such as: aposyndesis, local connectedness, arcwise disconnectedness, and contractibility.

Keywords: hyperspace; continuum; containment hyperspace; aposyndesis; finite graph; Peano continuum; contractibility
AMS Subject Classification: 54B15, 54B20, 54F15

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