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On the continuity of the elements of the Ellis semigroup and other properties

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Abstract: We consider discrete dynamical systems whose phase spaces are compact metrizable countable spaces. In the first part of the article, we study some properties that guarantee the continuity of all functions of the corresponding Ellis semigroup. For instance, if every accumulation point of X is fixed, we give a necessary and sufficient condition on a point $a \in X'$ in order that all functions of the Ellis semigroup $E(X, f)$ be continuous at the given point a. In the second part, we consider transitive dynamical systems. We show that if (X, f) is a transitive dynamical system and either every function of $E(X, f)$ is continuous or $|\omega_f(x)| = 1$ for each accumulation point x of X, then $E(X, f)$ is homeomorphic to X. Several examples are given to illustrate our results.

Keywords: discrete dynamical system; Ellis semigroup; p-iterate; p-limit point; ultrafilter; compact metric countable space
AMS Subject Classification: 54G20, 54D80

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