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On atomic ideals in some factor rings of $C(X, \mathbb{Z})$

Abstract: A nonzero R-module M is atomic if for each two nonzero elements a, b in M, both cyclic submodules Ra and Rb have nonzero isomorphic submodules. In this article it is shown that for an infinite P-space X, the factor rings $C(X, \mathbb{Z})/C_F(X, \mathbb{Z})$ and $C_c(X)/C_F(X)$ have no atomic ideals. This fact generalizes a result published in paper by A. Mozaffarikhah, E. Momtahan, A. R. Olfati and S. Safaeeyan (2020), which says that for an infinite set X, the factor ring $\mathbb{Z}^X/\mathbb{Z}^{(X)}$ has no atomic ideal. Another result is that for each infinite P-space X, the socle of the factor ring $C_c(X)/C_F(X)$ is always equal to zero. Also, zero-dimensional spaces X are characterized for which $C^F(X, \mathbb{Z})/C_F(X, \mathbb{Z})$ have atomic ideals.

Keywords: P-space; rings of integer-valued continuous functions; functionally countable subalgebra; atomic ideal; socle

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