

Marco L. A. Velásquez, André F. A. Ramalho,
Henrique F. de Lima, Márcio S. Santos, Arlandson M. S.
Oliveira

*Conformal Killing graphs in foliated Riemannian spaces with density:
rigidity and stability*

Comment.Math.Univ.Carolin. 62,2 (2021) 175–200.

Abstract: In this paper we investigate the geometry of conformal Killing graphs in a Riemannian manifold \overline{M}_f^{n+1} endowed with a weight function f and having a closed conformal Killing vector field V with conformal factor ψ_V , that is, graphs constructed through the flow generated by V and which are defined over an integral leaf of the foliation V^\perp orthogonal to V . For such graphs, we establish some rigidity results under appropriate constraints on the f -mean curvature. Afterwards, we obtain some stability results for f -minimal conformal Killing graphs of \overline{M}_f^{n+1} according to the behavior of ψ_V . Finally, related to conformal Killing graphs immersed in \overline{M}_f^{n+1} with constant f -mean curvature, we study the strong stability.

Keywords: weighted Riemannian manifold; conformal Killing graph; f -mean curvature; Bakry–Émery–Ricci tensor; strong f -stability

AMS Subject Classification: 53C42

REFERENCES

- [1] Aledo J. A., Rubio R. M., *Stable minimal surfaces in Riemannian warped products*, J. Geom. Anal. **27** (2017), no. 1, 65–78.
- [2] Alexandrov A. D., *Uniqueness theorems for surfaces in the large I*, Vestnik Leningrad Univ. **11** (1956), no. 19, 5–17 (Russian).
- [3] Alexandrov A. D., *A characteristic property of spheres*, Ann. Mat. Pura Appl. **58** (1962), no. 4, 303–315.
- [4] Alías L. J., Dajczer M., Ripoll J. R., *A Bernstein-type theorem for Riemannian manifolds with a Killing field*, Ann. Global Anal. Geom. **31** (2007), no. 4, 363–373.
- [5] Alías L. J., de Lira J. H. S., Malacarne J. M., *Constant higher-order mean curvature hypersurfaces in Riemannian spaces*, J. Inst. Math. Jussieu **5** (2006), no. 4, 527–562.
- [6] Bakry D., Émery M., *Diffusions hypercontractives*, Séminaire de probabilités, XIX, 1983/84, Lecture Notes in Math., 1123, Springer, Berlin, 1985, pages 177–206 (French).
- [7] Barbosa J. L. M., do Carmo M., Eschenburg J., *Stability of hypersurfaces with constant mean curvature in Riemannian manifolds*, Math. Z. **197** (1988), no. 1, 123–138.
- [8] Batista M., Cavalcante M. P., Pyo J., *Some isoperimetric inequalities and eigenvalue estimates in weighted manifolds*, J. Math. Anal. Appl. **419** (2014), no. 1, 617–626.
- [9] Bernstein S., *Sur les surfaces définies au moyen de leur courbure moyenne ou totale*, Ann. Sci. École Norm. Sup. **27** (1910), no. 3, 233–256 (French).
- [10] Caminha A., *The geometry of closed conformal vector fields on Riemannian spaces*, Bull. Braz. Math. Soc. (N.S.) **42** (2011), no. 2, 277–300.
- [11] Caminha A., de Lima H. F., *Complete vertical graphs with constant mean curvature in semi-Riemannian warped products*, Bull. Belg. Math. Soc. Simon Stevin **16** (2009), no. 1, 91–105.
- [12] Cañete A., Rosales C., *Compact stable hypersurfaces with free boundary in convex solid cones with homogeneous densities*, Cal. Var. Partial Differential Equations **51** (2014), no. 3–4, 887–913.
- [13] Castro K., Rosales C., *Free boundary stable hypersurfaces in manifolds with density and rigidity results*, J. Geom. Phys. **79** (2014), 14–28.
- [14] Cavalcante M. P., de Lima H. F., Santos M. S., *On Bernstein-type properties of complete hypersurfaces in weighted warped products*, Ann. Mat. Pura Appl. (4) **195** (2016), no. 2, 309–322.
- [15] Dajczer M., de Lira J. H., *Conformal Killing graphs with prescribed mean curvature*, J. Geom. Anal. **22** (2012), no. 3, 780–799.

- [16] Dajczer M., Hinojosa P., de Lira J. H., *Killing graphs with prescribed mean curvature*, Calc. Var. Partial Differential Equations **33** (2008), no. 2, 231–248.
- [17] de Lima H. F., de Lima J. R., Velásquez M. A. L., *On the nullity of conformal Killing graphs in foliated Riemannian spaces*, Aequationes Math. **87** (2014), no. 3, 285–299.
- [18] de Lima H. F., de Lima J. R., Velásquez M. A. L., *Entire conformal Killing graphs in foliated Riemannian spaces*, J. Geom. Anal. **25** (2015), no. 1, 171–188.
- [19] de Lima H. F., Oliveira A. M., Velásquez M. A. L., *On the uniqueness of complete two-sided hypersurfaces immersed in a class of weighted warped products*, J. Geom. Anal. **27** (2017), no. 3, 2278–2301.
- [20] Fang F., Li X.-D., Zhang Z., *Two generalizations of Cheeger–Gromoll splitting theorem via Bakry–Émery Ricci curvature*, Ann. Inst. Fourier (Grenoble) **59** (2009), no. 2, 563–573.
- [21] Hieu D. T., Nam T. L., *Bernstein type theorem for entire weighted minimal graphs in $\mathbb{G}^n \times \mathbb{R}$* , J. Geom. Phys. **81** (2014), 87–91.
- [22] Impera D., de Lira J. H., Pigola S., Setti A. G., *Height estimates for Killing graphs*, J. Geom. Anal. **28** (2018), no. 3, 2857–2885.
- [23] Impera D., Rimoldi M., *Stability properties and topology at infinity of f -minimal hypersurfaces*, Geom. Dedicata **178** (2015), 21–47.
- [24] Jellett J. J., *Sur la surface dont la courbure moyenne est constante*, J. Math. Pures Appl. **18** (1853), 163–167 (French).
- [25] Lichnerowicz A., *Variétés Riemanniennes à tenseur C non négatif*, C. R. Acad. Sci. Paris Sér. A-B **271** (1970), A650–A653 (French).
- [26] Lichnerowicz A., *Variétés Kähleriennes à première classe de Chern non négative et variétés Riemanniennes à courbure de Ricci généralisée non négative*, J. Differential Geometry **6** (1971/72), 47–94 (French).
- [27] Liebmann H., *Eine neue Eigenschaft der Kugel*, Nachr. Kg. Ges. Wiss. Göttingen, Math. Phys. Kl. (1899), 44–55 (German).
- [28] McGonagle M., Ross J., *The hyperplane is the only stable, smooth solution to the isoperimetric problem in Gaussian space*, Geom. Dedicata **178** (2015), 277–296.
- [29] Montiel S., *Unicity of constant mean curvature hypersurfaces in some Riemannian manifolds*, Indiana Univ. Math. J. **48** (1999), no. 2, 711–748.
- [30] O’Neill B., *Semi-Riemannian Geometry, With Applications to Relativity*, Pure and Applied Mathematics, 103, Academic Press, New York, 1983.
- [31] Pan T. K., *Conformal vector fields in compact Riemannian manifolds*, Proc. Amer. Math. Soc. **14** (1963), 653–657.
- [32] Rosales C., Cañete A., Bayle V., Morgan F., *On the isoperimetric problem in Euclidean space with density*, Calc. Var. Partial Differential Equations **31** (2008), no. 1, 27–46.
- [33] Wei G., Wylie W., *Comparison geometry for the Bakry–Émery Ricci tensor*, J. Differential Geom. **83** (2009), no. 2, 377–405.
- [34] Yau S. T., *Some function-theoretic properties of complete Riemannian manifold and their applications to geometry*, Indiana Univ. Math. J. **25** (1976), no. 7, 659–670.