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On a class of variational problems with linear growth and radial symmetry

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Abstract: We discuss variational problems on two-dimensional domains with energy densities of linear growth and with radially symmetric data. The smoothness of generalized minimizers is established under rather weak ellipticity assumptions. Further results concern the radial symmetry of solutions as well as a precise description of their behavior near the boundary.

Keywords: linear growth problem; symmetric solutions in 2D; existence of solutions in 2D; uniqueness solution in 2D; (non-)attainment of boundary data
AMS Subject Classification: 49J45, 49N60

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