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Classification of quasigroups according to directions of translations II

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Abstract: In each quasigroup Q there are defined six types of translations: the left, right and middle translations and their inverses. Two translations may coincide as permutations of Q, and yet be different when considered upon the web of the quasigroup. We shall call each of the translation types a direction and will associate it with one of the elements ι, l, r, s, ls and rs, i.e., the elements of a symmetric group S_3 . Properties of the directions are considered in part 1 of "Classification of quasigroups according to directions of translations I" by F. M. Sokhatsky and A. V. Lutsenko. Let $^{\sigma}\mathcal{M}$ denote the set of all translations of a direction $\sigma \in S_3$. The conditions $^{\sigma}\mathcal{M} = {}^{\kappa}\mathcal{M}$, where $\sigma, \kappa \in S_3$ and $\sigma \neq \kappa$, define nine quasigroup varieties. Four of them are well known: *LIP*, *RIP*, *MIP* and *CIP*. The remaining five quasigroup varieties are relatively new because they are left and right inverses of *CIP* variety and generalization of commutative, left and right symmetric quasigroups.

Keywords: quasigroup; parastrophe; parastrophic symmetry; parastrophic orbit; translation; direction; matrix quasigroup

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