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Rings whose nonsingular right modules are R-projective

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Abstract: A right R-module M is called R-projective provided that it is projective relative to the right R-module R_R. This paper deals with the rings whose all nonsingular right modules are R-projective. For a right nonsingular ring R, we prove that R_R is of finite Goldie rank and all nonsingular right R-modules are R-projective if and only if R is right finitely Σ-CS and flat right R-modules are R-projective. Then, R-projectivity of the class of nonsingular injective right modules is also considered. Over right nonsingular rings of finite right Goldie rank, it is shown that R-projectivity of nonsingular injective right modules is equivalent to R-projectivity of the injective hull $E(R_R)$. In this case, the injective hull $E(R_R)$ has the decomposition $E(R_R) = U_R \oplus V_R$, where U is projective and $\mathrm{Hom}(V, R/I) = 0$ for each right ideal I of R. Finally, we focus on the right orthogonal class $\mathcal{N}^{\perp}$ of the class $\mathcal{N}$ of nonsingular right modules.

Keywords: nonsingular module; R-projective module; flat module; perfect ring
AMS Subject Classification: 16D10, 16D40, 16D80, 16E30

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