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The (dis)connectedness of products of Hausdorff spaces in the box topology

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Abstract: In this paper the following two propositions are proved: (a) If X_α, $\alpha \in A$, is an infinite system of connected spaces such that infinitely many of them are nondegenerated completely Hausdorff topological spaces then the box product $\square_{\alpha \in A} X_\alpha$ can be decomposed into continuum many disjoint nonempty open subsets, in particular, it is disconnected. (b) If X_α, $\alpha \in A$, is an infinite system of Brown Hausdorff topological spaces then the box product $\square_{\alpha \in A} X_\alpha$ is also Brown Hausdorff, and hence, it is connected. A space is Brown if for every pair of its open nonempty subsets there exists a point common to their closures. There are many examples of countable Brown Hausdorff spaces in literature.

Keywords: box topology; connectedness; completely Hausdorff space; Urysohn space; Brown space
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References

[1] Acosta G., Madriz-Mendoza M., Dominguez J. D. C. A., *Brown spaces and the Golomb topology*, Open Acc. J. Math. Theor. Phy. **1** (2018), no. 6, 242–247.
[2] Bing R. H., *A connected countable Hausdorff space*, Proc. Amer. Math. Soc. **4** (1953), 474.
[3] Chatyrko V. A., Karassev A., *The (dis)connectedness of products in the box topology*, Questions Answers Gen. Topology **31** (2013), no. 1, 11–21.
[4] Chatyrko V. A., Nyagahakwa V., *Vitali selectors in topological groups and related semigroups of sets*, Questions Answers Gen. Topology **33** (2015), no. 2, 93–102.
[5] Golomb S. W., *A connected topology for integers*, Amer. Math. Monthly **66** 8 (1959), no. 8, 663–665.
[6] Halimskiĭ E. D., *The topologies of generalized segments*, Dokl. Akad. Nauk SSSR **189** (1969), 740–743.
[7] Hewitt E., *On two problems of Urysohn*, Ann. of Math. **47** (1946), no. 3, 503–509.
[8] Jones F. B., Stone A. H., *Countable locally connected Urysohn spaces*, Colloq. Math. **22** (1971), 239–244.
[9] Kannan V., Rajagopalan M., *On countable locally connected spaces*, Colloq. Math. **29** (1974), 93–100, 159.
[10] Kirch A. M., *A countable connected, locally connected Hausdorff space*, Amer. Math. Monthly **76** (1969), 169–171.
[11] Knight C. J., *Box topologies*, Quart. J. Math. Oxford Ser. (2) **15** (1964), 41–54.
[12] Kong T. Y., Kopperman R., Meyer P. R., *A topological approach to digital topology*, Amer. Math. Monthly **98** (1991), no. 10, 901–917.
[13] Lawrence L. B., *Infinite-dimensional countable connected Hausdorff spaces*, Houston J. Math. **20** (1994), no. 3, 539–546.
[14] Miller G. G., *Countable connected spaces*, Proc. Amer. Math. Soc. **26** (1970), 355–360.
[15] Munkres J. R., *Topology*, Prentice Hall, Upper Saddle River, 2000.
[16] Ritter G. X., *A connected locally connected, countable Hausdorff space*, Amer. Math. Monthly **83** (1976), no. 3, 185–186.
[17] Roy P., *A countable connected Urysohn space with a dispersion point*, Duke Math. J. **33** (1966), 331–333.
[18] Shimrat M., *Embedding in homogeneous spaces*, Quart. J. Math. Oxford Ser. (2) **5** (1954), 304–311.
[19] Vought E. J., *A countable connected Urysohn space with a dispersion point that is regular almost everywhere*, Colloq. Math. **28** (1973), 205–209.