Václav Kryštof

Further generalized versions of Ilmanen's lemma on insertion of $C^{1,\omega}$ or $C^{1,\omega}_{\mathrm{loc}}$ functions

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Abstract: The author proved in 2018 that if G is an open subset of a Hilbert space, $f_1, f_2 \colon G \to \mathbb{R}$ continuous functions and ω a nontrivial modulus such that $f_1 \leq f_2$, f_1 is locally semiconvex with modulus ω and f_2 is locally semiconcave with modulus ω, then there exists $f \in C^{1,\omega}_{\mathrm{loc}}(G)$ such that $f_1 \leq f \leq f_2$. This is a generalization of Ilmanen's lemma (which deals with linear modulus and functions on an open subset of $\mathbb{R}^n$). Here we extend the mentioned result from Hilbert spaces to some superreflexive spaces, in particular to L^p spaces, $p \in [2, \infty)$. We also prove a "global" version of Ilmanen's lemma (where a $C^{1,\omega}$ function is inserted between functions on an interval $I \subset \mathbb{R}$).

Keywords: Ilmanen's lemma; $C^{1,\omega}$ function; semiconvex function with general modulus
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