Bertrand Maillot, Christophe Chesneau

Generalized regression estimation for continuous time processes with values in functional spaces

Comment.Math.Univ.Carolin. 62,4 (2021) 461 –481.

Abstract: We consider two continuous time processes; the first one is valued in a semi-metric space, while the second one is real-valued. In some sense, we extend the results of F. Ferraty and P. Vieu in "Nonparametric models for functional data, with application in regression, time-series prediction and curve discrimination" (2004), by establishing the convergence, with rates, of the generalized regression function when a real-valued continuous time response is considered. As corollaries, we deduce the convergence of the conditional distribution function as well as conditional quantiles. Note that a parametric rate of convergence in probability is reached while working with a naive kernel.

Keywords: continuous time process; regression function estimation; conditional distribution function
AMS Subject Classification: 62G07, 62C05, 62E20

References

[1] Bosq D., *Vitesses optimales et superoptimales des estimateurs fonctionnels pour les processus à temps continu*, C. R. Acad. Sci. Paris Sér. I Math. **317** (1993), no. 11, 1075–1078 (French).

[2] Bosq D., *Nonparametric Statistics for Stochastic Processes*, Estimation and Prediction, Lecture Notes in Statistics, 110, Springer, New York, 1998.

[3] Bosq D., *Linear Processes in Function Spaces*, Theory and Applications, Lecture Notes in Statistics, 149, Springer, New York, 2000.

[4] Bradley R. C., *Basic properties of strong mixing conditions. A survey and some open questions*, Probab. Surv. **2** (2005), 107–144.

[5] Cardot H., Ferraty F., Sarda P., *Functional linear model*, Statist. Probab. Lett. **45** (1999), no. 1, 11–22.

[6] Cardot H., Ferraty F., Sarda P., *Spline estimators for the functional linear model*, Statist. Sinica **13** (2003), no. 3, 571–591.

[7] Castellana J. V., Leadbetter M. R., *On smoothed probability density estimation for stationary processes*, Stochastic Process. Appl. **21** (1986), no. 2, 179–193.

[8] Collomb G., Härdle W., *Strong uniform convergence rates in robust nonparametric time series analysis and prediction: kernel regression estimation from dependent observations*, Stochastic Process. Appl. **23** (1986), no. 1, 77–89.

[9] Dabo-Niang S., *Density estimation in a separable metric space*, Ann. I.S.U.P. **47** (2003), no. 1–2, 3–21.

[10] Dabo-Niang S., *Density estimation by orthogonal series in an infinite dimensional space: application to processes of diffusion type I*, The International Conf. on Recent Trends and Directions in Nonparametric Statistics, J. Nonparametr. Stat. **16** (2004), no. 1–2, 171–186.

[11] Demongeot J., Laksaci A., Madani F., Rachdi M., *A fast functional locally modeled conditional density and mode for functional time-series*, Recent Advances in Functional Data Analysis and Related Topics, Contrib. Statist., Physica Verlag, Springer, Heidelberg, 2011, 85–90.

[12] Ferraty F., Goia A., Vieu P., *Functional nonparametric model for time series: a fractal approach for dimension reduction*, Test **11** (2002), no. 2, 317–344.

[13] Ferraty F., Laksaci A., Tadj A., Vieu P., *Kernel regression with functional response*, Electron. J. Stat. **5** (2011), 159–171.

[14] Ferraty F., Rabhi A., Vieu P, *Conditional quantiles for dependent functional data with application to the climatic El Niño phenomenon*, Sankhyā **67** (2005), no. 2, 378–398.

[15] Ferraty F., Vieu P., *Nonparametric models for functional data, with application in regression, time-series prediction and curve discrimination*, The International Conf. on Recent Trends and Directions in Nonparametric Statistics, J. Nonparametr. Stat. **16** (2004), no. 1–2, 111–125.

[16] Ferraty F., Vieu P., *Nonparametric Functional Data Analysis*, Theory and Practice, Springer Series in Statistics, Springer, New York, 2006.

[17] Frank I. E., Friedman J. H., *A statistical view of some chemometric regression tools*, Technometrics **35** (1993), no. 2, 109–135.

[18] Grunig R., *Probabilités conditionnelles régulières sur des tribus de type non dénombrable*, Ann. Inst. H. Poincaré Sect. B (N.S.) **2** (1965/1966), no. 3, 227–229 (French).

[19] Jiřina M., *Conditional probabilities on strictly separable σ-algebras*, Czechoslovak. Math. J. **4(79)** (1954), 372–380.

[20] Jiřina M., *On regular conditional probabilities*, Czechoslovak. Math. J. **9(84)** (1959), 445–451.

[21] Krzyżak A., Pawlak M., *The pointwise rate of convergence of the kernel regression estimate*, J. Statist. Plann. Inference **16** (1987), no. 2, 159–166.

[22] Masry E., *Nonparametric regression estimation for dependent functional data: asymptotic normality*, Stochastic Process. Appl. **115** (2005), no. 1, 155–177.

[23] Ramsay J. O., Silverman B. W., *Functional Data Analysis*, Springer Series in Statistics, Springer, New York, 2005.

[24] Rio E., *Théorie asymptotique des processus aléatoires faiblement dépendants*, Mathématiques & Applications, 31, Springer, Berlin, 2000 (French).

[25] Rosenblatt M., *Conditional probability density and regression estimators*, in Multivariate Analysis II, Proc. Second Internat. Sympos., Dayton, Ohio, 1968, Academic Press, New York, 1969, pages 25–31.

[26] Roussas G. G., *Nonparametric regression estimation under mixing conditions*, Stochastic Process. Appl. **36** (1990), no. 1, 107–116.

[27] Stone C. J., *Optimal global rates of convergence for nonparametric regression*, Ann. Statist. **10** (1982), no. 4, 1040–1053.

[28] Watson G. S., *Smooth regression analysis*, Sankhyā Ser. A **26** (1964), 359–372.