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Seeking a network characterization of Corson compacta

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Abstract: We say that a collection $\mathcal{A}$ of subsets of X has property (CC) if there is a set D and point-countable collections $\mathcal{C}$ of closed subsets of X such that for any $A \in \mathcal{A}$ there is a finite subcollection $\mathcal{F}$ of $\mathcal{C}$ such that $A = D \setminus \bigcup \mathcal{F}$. Then we prove that any compact space is Corson if and only if it has a point-σ-(CC) base. A characterization of Corson compacta in terms of (strong) point network is also given. This provides an answer to an open question in "A Biased View of Topology as a Tool in Functional Analysis" (2014) by B. Cascales and J. Orihuela and as in "Network characterization of Gul'ko compact spaces and their relatives" (2004) by F. Garcia, L. Oncina, J. Orihuela, which asked whether there is a network characterization of the class of Corson compacta.

Keywords: Corson compacta; point network; condition (F); almost subbase; additively $\aleph_0$-Noetherian
AMS Subject Classification: 54D30, 46B50

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