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*Linear operator identities in quasigroups*

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**Abstract:** We study identities of the form

$$L_{x_0}\varphi_1 \cdots \varphi_n R_{x_{n+1}} = R_{x_{n+1}}\varphi_{\sigma(1)} \cdots \varphi_{\sigma(n)}L_{x_0}$$

in quasigroups, where  $n \geq 1$ ,  $\sigma$  is a permutation of  $\{1, \dots, n\}$ , and for each  $i$ ,  $\varphi_i$  is either  $L_{x_i}$  or  $R_{x_i}$ . We prove that in a quasigroup, every such identity implies commutativity. Moreover, if  $\sigma$  is chosen randomly and uniformly, it also satisfies associativity with probability approaching 1 as  $n \rightarrow \infty$ .

**Keywords:** quasigroup; linear identity; associativity; commutativity

**AMS Subject Classification:** 05C78

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