Lhoussain El Fadil

On power integral bases for certain pure number fields defined by $x^{18}-m$

Comment.Math.Univ.Carolin. 63,1 (2022) 11 –19.

Abstract: Let $K=\mathbb{Q}(\alpha)$ be a number field generated by a complex root α of a monic irreducible polynomial $f(x)=x^{18}-m$, $m\neq\mp1$, is a square free rational integer. We prove that if $m\equiv 2$ or 3 (mod 4) and $m\not\equiv\mp1$ (mod 9), then the number field K is monogenic. If $m\equiv 1$ (mod 4) or $m\equiv 1$ (mod 9), then the number field K is not monogenic.

Keywords: power integral base; theorem of Ore; prime ideal factorization
AMS Subject Classification: 11R04, 11R16, 11R21

References

[1] Ahmad S., Nakahara T., Hameed A., *On certain pure sextic fields related to a problem of Hasse*, Int. J. Algebra Comput. **26** (2016), no. 3, 577–583.
[2] Ahmad S., Nakahara T., Husnine S. M., *Power integral bases for certain pure sextic fields*, Int. J. Number Theory **10** (2014), no. 8, 2257–2265.
[3] El Fadil L., *Computation of a power integral basis of a pure cubic number field*, Int. J. Contemp. Math. Sci. **2** (2007), no. 13–16, 601–606.
[4] El Fadil L., *On Newton polygons techniques and factorization of polynomials over Henselian fields*, J. Algebra Appl. **19** (2020), no. 10, 2050188, 9 pages.
[5] El Fadil L., *On power integral bases for certain pure number fields defined by $x^{24}-m$*, Studia Sci. Math. Hungar. **57** (2020), no. 3, 397–407.
[6] El Fadil L., *On power integral bases for certain pure number fields*, Publ. Math. Debrecen **100** (2022), no. 1–2, 219–231.
[7] El Fadil L., Montes J., Nart E., *Newton polygons and p-integral bases of quartic number fields*, J. Algebra Appl. **11** (2012), no. 4, 1250073, 33 pages.
[8] Funakura T., *On integral bases of pure quartic fields*, Math. J. Okayama Univ. **26** (1984), 27–41.
[9] Gaál I., *Power integral bases in algebraic number fields*, Ann. Univ. Sci. Budapest. Sect. Comput. **18** (1999), 61–87.
[10] Gaál I., *Diophantine Equations and Power Integral Bases*, Theory and Algorithm, Birkhäuser/Springer, Cham, 2019.
[11] Gaál I., Olajos P., Pohst M., *Power integral bases in orders of composite fields*, Experiment. Math. **11** (2002), no. 1, 87–90.
[12] Gaál I., Remete L., *Binomial Thue equations and power integral bases in pure quartic fields*, J. Algebra Number Theory Appl. **32** (2014), no. 1, 49–61.
[13] Gaál I., Remete L., *Integral bases and monogenity of pure fields*, J. Number Theory **173** (2017), 129–146.
[14] Hameed A., Nakahara T., *Integral bases and relative monogenity of pure octic fields*, Bull. Math. Soc. Sci. Math. Roumanie (N.S.) **58(106)** (2015), no. 4, 419–433.
[15] Ore Ö., *Newtonsche Polygone in der Theorie der algebraischen Körper*, Math. Ann. **99** (1928), no. 1, 84–117 (German).
[16] Pethő A., Pohst M., *On the indices of multiquadratic number fields*, Acta Arith. **153** (2012), no. 4, 393–414.