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On the asymptotics of counting functions for Ahlfors regular sets

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Abstract: We deal with the so-called Ahlfors regular sets (also known as s-regular sets) in metric spaces. First we show that those sets correspond to a certain class of tree-like structures. Building on this observation we then study the following question: Under which conditions does the limit $\lim_{\varepsilon\to 0+} \varepsilon^s N(\varepsilon, K)$ exist, where K is an s-regular set and $N(\varepsilon, K)$ is for instance the ε-packing number of K?

Keywords: Ahlfors regular; s-regular; packing number; Minkowski measurability; renewal theory

AMS Subject Classification: 30L99, 28A80

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