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Totally Brown subsets of the Golomb space and the Kirch space

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Abstract: A topological space X is totally Brown if for each $n \in \mathbb{N} \setminus \{1\}$ and every nonempty open subsets $U_1, U_2, \ldots, U_n$ of X we have $\mathrm{cl}_X(U_1) \cap \mathrm{cl}_X(U_2) \cap \cdots \cap \mathrm{cl}_X(U_n) \neq \emptyset$. Totally Brown spaces are connected. In this paper we consider the Golomb topology τ_G on the set $\mathbb{N}$ of natural numbers, as well as the Kirch topology τ_K on $\mathbb{N}$. Then we examine subsets of these spaces which are totally Brown. Among other results, we characterize the arithmetic progressions which are either totally Brown or totally separated in $(\mathbb{N}, \tau_G)$. We also show that $(\mathbb{N}, \tau_G)$ and $(\mathbb{N}, \tau_K)$ are aposyndetic. Our results generalize properties obtained by A. M. Kirch in 1969 and by P. Szczuka in 2010, 2013 and 2015.

Keywords: arithmetic progression; Golomb topology; Kirch topology; totally Brown space; totally separated space
AMS Subject Classification: 11B25, 54D05, 11A41, 11B05, 54A05, 54D10

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