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Self-small products of abelian groups

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Abstract: Let A and B be two abelian groups. The group A is called B-small if the covariant functor $\mathrm{Hom}(A,-)$ commutes with all direct sums $B^{(\kappa)}$ and A is self-small provided it is A-small. The paper characterizes self-small products applying developed closure properties of the classes of relatively small groups. As a consequence, self-small products of finitely generated abelian groups are described.

Keywords: self-small abelian group; slender group
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