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Selectors of discrete coarse spaces

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Abstract: Given a coarse space (X, \mathcal{E}) with the bornology \mathcal{B} of bounded subsets, we extend the coarse structure \mathcal{E} from $X \times X$ to the natural coarse structure on $(\mathcal{B} \setminus \{\emptyset\}) \times (\mathcal{B} \setminus \{\emptyset\})$ and say that a macro-uniform mapping $f: (\mathcal{B} \setminus \{\emptyset\}) \rightarrow X$ (or $f: [X]^2 \rightarrow X$) is a selector (or 2-selector) of (X, \mathcal{E}) if $f(A) \in A$ for each $A \in \mathcal{B} \setminus \{\emptyset\}$ ($A \in [X]^2$, respectively). We prove that a discrete coarse space (X, \mathcal{E}) admits a selector if and only if (X, \mathcal{E}) admits a 2-selector if and only if there exists a linear order “ \leq ” on X such that the family of intervals $\{[a, b] : a, b \in X, a \leq b\}$ is a base for the bornology \mathcal{B} .

Keywords: bornology; coarse space; selector

AMS Subject Classification: 54C65

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