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Mersenne numbers as a difference of two Lucas numbers

Comment.Math.Univ.Carolin. 63,3 (2022) 269 –276.

Abstract: Let $(L_n)_{n\geq 0}$ be the Lucas sequence. We show that the Diophantine equation $L_n - L_m = M_k$ has only the nonnegative integer solutions $(n, m, k) = (2, 0, 1)$, $(3, 1, 2)$, $(3, 2, 1)$, $(4, 3, 2)$, $(5, 3, 3)$, $(6, 2, 4)$, $(6, 5, 3)$ where $M_k = 2^k - 1$ is the kth Mersenne number and $n > m$.

Keywords: Lucas number; Mersenne number; Diophantine equation; linear forms in logarithm
AMS Subject Classification: 11B39, 11J86, 11D61

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