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The common division topology on $\mathbb{N}$

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Abstract: A topological space X is totally Brown if for each $n \in \mathbb{N} \setminus \{1\}$ and every nonempty open subsets $U_1, U_2, \dots, U_n$ of X we have $\mathrm{cl}_X(U_1) \cap \mathrm{cl}_X(U_2) \cap \cdots \cap \mathrm{cl}_X(U_n) \neq \emptyset$. Totally Brown spaces are connected. In this paper we consider a topology τ_S on the set $\mathbb{N}$ of natural numbers. We then present properties of the topological space $(\mathbb{N}, \tau_S)$, some of them involve the closure of a set with respect to this topology, while others describe subsets which are either totally Brown or totally separated. Our theorems generalize results proved by P. Szczuka in 2013, 2014, 2016 and by P. Szyszkowska and M. Szyszkowski in 2018.

Keywords: arithmetic progression; common division topology; totally Brown space; totally separated space

AMS Subject Classification: 11B25, 54D05, 11A41, 11B05, 54A05, 54D10

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