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*Hyperspace selections avoiding points*

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**Abstract:** We deal with a hyperspace selection problem in the setting of connected spaces. We present two solutions of this problem illustrating the difference between selections for the nonempty closed sets, and those for the at most two-point sets. In the first case, we obtain a characterisation of compact orderable spaces. In the latter case — that of selections for at most two-point sets, the same selection property is equivalent to the existence of a ternary relation on the space, known as a cyclic order, and gives a characterisation of the so called weakly cyclically orderable spaces.

**Keywords:** Vietoris topology; continuous selection; weak selection; weakly orderable space; weakly cyclically orderable space

**AMS Subject Classification:** 54B20, 54C65, 54D05, 54D30, 54F05, 54F65

REFERENCES

- [1] Brouwer A. E., *A characterization of connected (weakly) orderable spaces*, Math. Centrum, Amsterdam, Afd. zuivere Wisk. ZW 10/71 (1971), 7 pages.
- [2] Brouwer A. E., *On the topological characterization of the real line*, Math. Centrum, Amsterdam, Afd. zuivere Wisk. ZW 8/71 (1971), 6 pages.
- [3] Buhagiar D., Gutev V., *Selections and deleted symmetric products*, Tsukuba J. Math. **41** (2017), no. 1, 1–20.
- [4] Čech E., *Topological Spaces*, Publishing House of the Czechoslovak Academy of Sciences, Praha, Interscience Publishers John Wiley & Sons, London, 1966.
- [5] Čoban M. M., *Many-valued mappings and Borel sets*, Trans. Mosc. Math. Soc. **22** (1970), 258–280.
- [6] van Dalen J., Wattel E., *A topological characterization of ordered spaces*, General Topology and Appl. **3** (1973), 347–354.
- [7] Duda R., *On ordered topological spaces*, Fund. Math. **63** (1968), 295–309.
- [8] Eilenberg S., *Ordered topological spaces*, Amer. J. Math. **63** (1941), 39–45.
- [9] Engelking R., Heath R. W., Michael E., *Topological well-ordering and continuous selections*, Invent. Math. **6** (1968), 150–158.
- [10] García-Ferreira S., Gutev V., Nogura T., Sanchis M., Tomita A., *Extreme selections for hyperspaces of topological spaces*, Proc. of the International Conf. on Topology and Its Applications, Yokohama, 1999, Topology Appl. **122** (2002), no. 1–2, 157–181.
- [11] Gutev V., *Weak orderability of second countable spaces*, Fund. Math. **196** (2007), no. 3, 275–287.
- [12] Gutev V., *Selections and hyperspaces*, Recent Progress in General Topology, III, Atlantis Press, Paris, 2014, pages 535–579.
- [13] Gutev V., *Selections and approaching points in products*, Comment. Math. Univ. Carolin. **57** (2016), no. 1, 89–95.
- [14] Gutev V., *Scattered spaces and selections*, Topology Appl. **231** (2017), 306–315.
- [15] Gutev V., Nogura T., *Selections and order-like relations*, Appl. Gen. Topol. **2** (2001), no. 2, 205–218.
- [16] Gutev V., Nogura T., *Fell continuous selections and topologically well-orderable spaces*, Mathematika **51** (2004), no. 1–2, 163–169.
- [17] Gutev V., Nogura T., *Set-maximal selections*, Topology Appl. **157** (2010), no. 1, 53–61.
- [18] Gutev V., Nogura T., *Weak orderability of topological spaces*, Topology Appl. **157** (2010), no. 8, 1249–1274.
- [19] Hocking J. G., Young G. S., *Topology*, Addison–Wesley Publishing, London, 1961.
- [20] Huntington E. V., *A set of independent postulates for cyclic order*, Proc. Natl. Acad. Sci. USA **2** (1916), 630–631.
- [21] Huntington E. V., *Sets of completely independent postulates for cyclic order*, Proc. Natl. Acad. Sci. USA **10** (1924), 74–78.

- [22] Kok H., *Connected Orderable Spaces*, Mathematical Centre Tracts, 49, Mathematisch Centrum, Amsterdam, 1973.
- [23] Michael E., *Topologies on spaces of subsets*, Trans. Amer. Math. Soc. **71** (1951), 152–182.
- [24] van Mill J., Wattel E., *Selections and orderability*, Proc. Amer. Math. Soc. **83** (1981), no. 3, 601–605.
- [25] Nadler S. B., Jr., *Continuum Theory*, An Introduction, Monographs and Textbooks in Pure and Applied Mathematics, 158, Marcel Dekker, New York, 1992.
- [26] Nogura T., Shakhmatov D., *Characterizations of intervals via continuous selections*, Rend. Circ. Mat. Palermo (2) **46** (1997), no. 2, 317–328.
- [27] Whyburn G. T., *Concerning the cut points of continua*, Trans. Amer. Math. Soc. **30** (1928), no. 3, 597–609.
- [28] Willard S., *General Topology*, Addison–Wesley Publishing, London, 1970.