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The category of compactifications and its coreflections

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Abstract: We define "the category of compactifications", which is denoted **CM**, and consider its family of coreflections, denoted **corCM**. We show that **corCM** is a complete lattice with bottom the identity and top an interpretation of the Čech–Stone β. A $c \in$**corCM** implies the assignment to each locally compact, noncompact Y a compactification minimum for membership in the "object-range" of c. We describe the minimum proper compactifications of locally compact, noncompact spaces, show that these generate the atoms in **corCM** (thus **corCM** is not a set), show that any $c \in$**corCM** not the identity is above an atom, and that β is not the supremum of atoms.

Keywords: compactification; coreflection; atom in a lattice
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