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Some results on derangement polynomials

Abstract: We study moments of the difference $D_n(x) - x^n n!\, \mathrm{e}^{-1/x}$ concerning derangement polynomials $D_n(x)$. For the first moment, we obtain an explicit formula in terms of the exponential integral function and we show that it is always negative for $x > 0$. For the higher moments, we obtain a multiple integral representation of the order of the moment under computation.

Keywords: derangement; permutation; integration
AMS Subject Classification: 05A05, 05A16, 26A06

References

[1] Aigner M., Ziegler G. M., *Proofs from The Book*, Springer, Berlin, 2018.

[2] Askey R. A., Ismail M. E. H., *Permutation problems and special functions*, Canadian. J. Math. **28** (1976), no. 4, 853–874.

[3] Axler S., *Measure, Integration & Real Analysis*, Graduate Texts in Mathematics, 282, Springer, Cham, 2020.

[4] Benyattou A., *Derangement polynomials with a complex variable*, Notes Number Theory Discrete Math. **26** (2020), no. 4, 128–135.

[5] Chow C.-O., *On derangement polynomials of type B*, Sém. Lothar. Combin. **55** (2005/07), Art. B55b, 6 pages.

[6] Chow C.-O., *On derangement polynomials of type B. II*, J. Combin. Theory Ser. A **116** (2009), no. 4, 816–830.

[7] Hassani M., *Derangements and applications*, J. Integer Seq. **6** (2003), no. 1, Art. 03.1.2, 8 pages.

[8] Hassani M., *Cycles in graphs and derangements*, Math. Gaz. **88** (2004), no. 511, 123–126.

[9] Hassani M., *Enumeration by* e, Modern Discrete Mathematics and Analysis: Springer Optim. Appl., 131, Springer, Cham, 2018, pages 227–233.

[10] Hassani M., *Derangements and alternating sum of permutations by integration*, J. Integer Seq. **23** (2020), no. 7, Art. 20.7.8, 9 pages.

[11] Hassani M., *On a difference concerning the number* e *and summation identities of permutations*, J. Inequal. Spec. Funct. **12** (2021), no. 1, 14–22.

[12] Kayll P. M., *Integrals don't have anything to do with discrete math, do they?*, Math. Mag. **84** (2011), no. 2, 108–119.

[13] LeVeque W. J., *Topics in Number Theory. Vols. 1 and 2*, Addison–Wesley Publishing, Mass, 1956.

[14] Radoux C., *Déterminant de Hankel construit sur des polynômes liés aux nombres de dérangements*, European J. Combin. **12** (1991), no. 4, 327–329 (French).

[15] Radoux C., *Addition formulas for polynomials built on classical combinatorial sequences*, Proc. of the 8th International Congress on Computational and Applied Mathematics, J. Comput. Appl. Math. **115** (2000), no. 1–2, 471–477.