

## Zbigniew Lipecki

*Order intervals in  $C(K)$ . Compactness, coincidence of topologies, metrizability*

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**Abstract:** Let  $K$  be a compact space and let  $C(K)$  be the Banach lattice of real-valued continuous functions on  $K$ . We establish eleven conditions equivalent to the strong compactness of the order interval  $[0, x]$  in  $C(K)$ , including the following ones: (i)  $\{x > 0\}$  consists of isolated points of  $K$ ; (ii)  $[0, x]$  is pointwise compact; (iii)  $[0, x]$  is weakly compact; (iv) the strong topology and that of pointwise convergence coincide on  $[0, x]$ ; (v) the strong and weak topologies coincide on  $[0, x]$ . Moreover, the weak topology and that of pointwise convergence coincide on  $[0, x]$  if and only if  $\{x > 0\}$  is scattered. Finally, the weak topology on  $[0, x]$  is metrizable if and only if the topology of pointwise convergence on  $[0, x]$  is such if and only if  $\{x > 0\}$  is countable.

**Keywords:** real linear lattice; order interval; locally solid; Banach lattice  $C(K)$ ; strongly compact; weakly compact; pointwise compact; coincidence of topologies; metrizable; scattered; Čech–Stone compactification

**AMS Subject Classification:** 46A40, 46B42, 46E05, 54C35, 54D30

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