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Counterexamples to Hedetniemi's conjecture and infinite Boolean lattices

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Abstract: We prove that for any $c \geq 5$, there exists an infinite family $(G_n)_{n \in \mathbb{N}}$ of graphs such that $\chi(G_n) > c$ for all $n \in \mathbb{N}$ and $\chi(G_m \times G_n) \leq c$ for all $m \neq n$. These counterexamples to Hedetniemi's conjecture show that the Boolean lattice of exponential graphs with K_c as a base is infinite for $c \geq 5$.

Keywords: categorical product; graph colouring; Hedetniemi's conjecture

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REFERENCES

- [1] Alishahi M., Hajiabolhassan H., *Alternating number of categorical product of graphs*, Discrete Math. **341** (2018), no. 5, 1316–1324.
- [2] Baum S., Stiebitz M., *Coloring of Graphs without Short Odd Paths between Vertices of the Same Color Class*, Syddansk Universitet, Odense, 2005.
- [3] Duffus D., Sauer N., *Lattices arising in categorical investigations of Hedetniemi's conjecture*, Discrete Math. **152** (1996), no. 1–3, 125–139.
- [4] El-Zahar M., Sauer N.W., *The chromatic number of the product of two 4-chromatic graphs is 4*, Combinatorica **5** (1985), no. 2, 121–126.
- [5] Godsil C., Roberson D.E., Šámal R., Severini S., *Sabidussi versus Hedetniemi for three variations of the chromatic number*, Combinatorica **36** (2016), no. 4, 395–415.
- [6] Gyárfás A., Jensen T., Stiebitz M., *On graphs with strongly independent color-classes*, J. Graph Theory **46** (2004), no. 1, 1–14.
- [7] Hahn G., Tardif C., *Graph homomorphisms: structure and symmetry*, Graph symmetry, Montreal, 1996, NATO Adv. Sci. Inst. Ser. C: Math. Phys. Sci., 497, Kluwer Acad. Publ., Dordrecht, 1997, pages 107–166.
- [8] Hajiabolhassan H., *On colorings of graph powers*, Discrete Math. **309** (2009), no. 13, 4299–4305.
- [9] Hajiabolhassan H., Taherkhani A., *Graph powers and graph homomorphisms*, Electron. J. Combin. **17** (2010), no. 1, Research Paper 17, 16 pages.
- [10] He X., Wigderson Y., *Hedetniemi's conjecture is asymptotically false*, J. Combin. Theory Ser. B **146** (2021), 485–494.
- [11] Hedetniemi S. T., *Homomorphisms of Graphs and Automata*, Thesis Ph.D., University of Michigan, Michigan, 1966.
- [12] Shitov Y., *Counterexamples to Hedetniemi's conjecture*, Ann. of Math. (2) **190** (2019), no. 2, 663–667.
- [13] Simonyi G., Tardos G., *Local chromatic number, Ky Fan's theorem and circular colorings*, Combinatorica **26** (2006), no. 5, 587–626.
- [14] Simonyi G., Zsbán A., *On topological relaxations of chromatic conjectures*, European J. Combin. **31** (2010), no. 8, 2110–2119.
- [15] Tardif C., *Hedetniemi's conjecture and dense Boolean lattices*, Order **28** (2011), no. 2, 181–191.
- [16] Tardif C., *The chromatic number of the product of 14-chromatic graphs can be 13*, Combinatorica **42** (2022), no. 2, 301–308.
- [17] Tardif C., Zhu X., *The level of nonmultiplicativity of graphs*, Algebraic and topological methods in graph theory, Discrete Math. **244** (2002), no. 1–3, 461–471.
- [18] Tardif C., Zhu X., *A note on Hedetniemi's conjecture, Stahl's conjecture and the Poljak–Rödl function*, Electron. J. Combin. **26** (2019), no. 4, Paper No. 4.32, 5 pages.
- [19] Wrochna M., *On inverse powers of graphs and topological implications of Hedetniemi's conjecture*, J. Combin. Theory Ser. B **139** (2019), 267–295.
- [20] Wrochna M., *Smaller counterexamples to Hedetniemi's conjecture*, available at arXiv:2012.13558 [math.CO] (2020), 9 pages.
- [21] Zhu X., *A survey on Hedetniemi's conjecture*, Taiwanese J. Math. **2** (1998), no. 1, 1–24.

- [22] Zhu X., *The fractional version of Hedetniemi's conjecture is true*, European J. Combin. **32** (2011), no. 7, 1168–1175.
- [23] Zhu X., *A note on the Poljak–Rödl function*, Electron. J. Combin. **27** (2020), no. 3, Paper No. 3.2, 4 pages.
- [24] Zhu X., *Relatively small counterexamples to Hedetniemi's conjecture*, J. Combin. Theory Ser. B **146** (2021), 141–150.