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Exponential separability is preserved by some products

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Abstract: We show that exponential separability is an inverse invariant of closed maps with countably compact exponentially separable fibers. This implies that it is preserved by products with a scattered compact factor and in the products of sequential countably compact spaces. We also provide an example of a σ-compact crowded space in which all countable subspaces are scattered. If X is a Lindelöf space and every $Y \subset X$ with $|Y| \le 2^{\omega_1}$ is scattered, then X is functionally countable; if every $Y \subset X$ with $|Y| \le 2^{\mathfrak{c}}$ is scattered, then X is exponentially separable. A Lindelöf Σ-space X must be exponentially separable provided that every $Y \subset X$ with $|Y| \le \mathfrak{c}$ is scattered. Under the Luzin axiom ($2^{\omega_1} > \mathfrak{c}$) we characterize weak exponential separability of $C_p(X,[0,1])$ for any metrizable space X. Our results solve several published open questions.

Keywords: Lindelöf space; scattered space; σ-product; function space; P-space; exponentially separable space; product; functionally countable space; weakly exponentially separable space
AMS Subject Classification: 54G12, 54G10, 54C35, 54D65

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