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Degree polynomial for vertices in a graph and its behavior under graph operations

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Abstract: We introduce a new concept namely the degree polynomial for the vertices of a simple graph. This notion leads to a concept, namely, the degree polynomial sequence which is stronger than the concept of degree sequence. After obtaining the degree polynomial sequence for some well-known graphs, we prove a theorem which gives a necessary condition for the realizability of a sequence of polynomials with positive integer coefficients. Also we calculate the degree polynomial for the vertices of the join, Cartesian product, tensor product, and lexicographic product of two simple graphs and for the vertices of the complement of a simple graph. Some examples, counterexamples, and open problems concerning these subjects is given as well.

Keywords: degree polynomial; degree polynomial sequence; degree sequence; graph operation
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