

# Jing Lu, Bin Zhao, Kaiyun Wang, Dongsheng Zhao

## *Quasicontinuous spaces*

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**Abstract:** We lift the notion of quasicontinuous posets to the topology context, called quasicontinuous spaces, and further study such spaces. The main results are: (1) A  $T_0$  space  $(X, \tau)$  is a quasicontinuous space if and only if  $SI(X)$  is locally hypercompact if and only if  $(\tau_{SI}, \subseteq)$  is a hypercontinuous lattice; (2) a  $T_0$  space  $X$  is an  $SI$ -continuous space if and only if  $X$  is a meet continuous and quasicontinuous space; (3) if a  $C$ -space  $X$  is a well-filtered poset under its specialization order, then  $X$  is a quasicontinuous space if and only if it is a quasicontinuous domain under the specialization order; (4) there exists an adjunction between the category of quasicontinuous domains and the category of quasicontinuous spaces which are well-filtered posets under their specialization orders.

**Keywords:** quasicontinuous space; hypercontinuous lattice;  $SI$ -continuous space; locally hypercompact space; meet continuous space

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