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Hyperplanes in matroids and the axiom of choice

Abstract: We show that in set theory without the axiom of choice ZF, the statement sH: "Every proper closed subset of a finitary matroid is the intersection of hyperplanes including it" implies AC^{fin}, the axiom of choice for (nonempty) finite sets. We also provide an equivalent of the statement AC^{fin} in terms of "graphic" matroids. Several open questions stay open in ZF, for example: does sH imply the axiom of choice?

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