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Maximal independent sets, variants of chain/antichain principle and cofinal subsets without AC

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Abstract: In set theory without the axiom of choice (AC), we observe new relations of the following statements with weak choice principles.

- ∘ $\mathcal{P}_{\rm lf,c}$ (Every locally finite connected graph has a maximal independent set).
- ∘ $\mathcal{P}_{\rm lc,c}$ (Every locally countable connected graph has a maximal independent set).
- ∘ $\mathrm{CAC}_1^{\aleph_\alpha}$ (If in a partially ordered set all antichains are finite and all chains have size $\aleph_\alpha$, then the set has size $\aleph_\alpha$) if $\aleph_\alpha$ is regular.
- ∘ CWF (Every partially ordered set has a cofinal well-founded subset).
- ∘ $\mathcal{P}_{G,H_2}$ (For any infinite graph $G = (V_G, E_G)$ and any finite graph $H = (V_H, E_H)$ on 2 vertices, if every finite subgraph of G has a homomorphism into H, then so has G).
- ∘ If $G = (V_G, E_G)$ is a connected locally finite chordal graph, then there is an ordering "$<$" of V_G such that $\{w < v \colon \{w, v\} \in E_G\}$ is a clique for each $v \in V_G$.

Keywords: variants of chain/antichain principle; graph homomorphism; maximal independent sets; cofinal well-founded subsets of partially ordered sets; axiom of choice; Fraenkel–Mostowski (FM) permutation models of ZFA + ¬ AC

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