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On extensions of families of operators

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Abstract: The strong closure of feasible states of families of operators is studied. The results are obtained for self-adjoint operators in reflexive Banach spaces and for more concrete case - families of elliptic systems encountered in the optimal layout of r materials. The results show when it is possible to parametrize the strong closure by the same type of operators. The results for systems of elliptic operators for the case when number of unknown functions m is less than the dimension n of the reference domain are well-known, but we present several different approaches in this paper to prove that parametrization of the strong closure of feasible states can be done by convexification. Also, a new approach is offered to prove result for the strong closure of cogradients. There are given counterexamples for the case $m \geq n$ when the parametrization by convexification is not possible. This extends the known result for the case $m = n = 2$.

Keywords: strong closure; feasible state; operator; elliptic system
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