

Alan Dow, Saharon Shelah

On the bounding, splitting, and distributivity numbers

Comment.Math.Univ.Carolin. 64,3 (2023) 331–351.

Abstract: The cardinal invariants $\mathfrak{h}, \mathfrak{b}, \mathfrak{s}$ of $\mathcal{P}(\omega)$ are known to satisfy that $\omega_1 \leq \mathfrak{h} \leq \min\{\mathfrak{b}, \mathfrak{s}\}$. We prove that all inequalities can be strict. We also introduce a new upper bound for \mathfrak{h} and show that it can be less than \mathfrak{s} . The key method is to utilize finite support matrix iterations of ccc posets following paper Ultrafilters with small generating sets by A. Blass and S. Shelah (1989).

Keywords: cardinal invariants of the continuum; matrix forcing

AMS Subject Classification: 03E15

REFERENCES

- [1] Balcar B., Pelant J., Simon P., *The space of ultrafilters on \mathbb{N} covered by nowhere dense sets*, Fund. Math. **110** (1980), no. 1, pages 11–24.
- [2] Baumgartner J. E., Dordal P., *Adjoining dominating functions*, J. Symbolic Logic **50** (1985), no. 1, 94–101.
- [3] Blass A., *Applications of superperfect forcing and its relatives*, Conf. Set Theory and Its Applications, Toronto, 1987, Lecture Notes in Math., 1401, Springer, Berlin, 1989, pages 18–40.
- [4] Blass A., Shelah S., *Ultrafilters with small generating sets*, Israel J. Math. **65** (1989), no. 3, 259–271.
- [5] Brendle J., Fischer V., *Mad families, splitting families and large continuum*, J. Symbolic Logic **76** (2011), no. 1, 198–208.
- [6] Brendle J., Raghavan D., *Bounding, splitting, and almost disjointness*, Ann. Pure Appl. Logic **165** (2014), no. 2, 631–651.
- [7] Dow A., Shelah S., *On the cofinality of the splitting number*, Indag. Math. (N.S.) **29** (2018), no. 1, 382–395.
- [8] Dow A., Shelah S., *Pseudo P-points and splitting number*, Arch. Math. Logic **58** (2019), no. 7–8, 1005–1027.
- [9] Fischer V., Friedman S. D., Mejía D. A., Montoya D. C., *Coherent systems of finite support iterations*, J. Symb. Log. **83** (2018), no. 1, 208–236.
- [10] Fischer V., Mejia D. A., *Splitting, bounding, and almost disjointness can be quite different*, Canad. J. Math. **69** (2017), no. 3, 502–531.
- [11] Fischer V., Steprāns J., *The consistency of $\mathfrak{b} = \kappa$ and $\mathfrak{s} = \kappa^+$* , Fund. Math. **201** (2008), no. 3, 283–293.
- [12] Goldstern M., Kellner J., Mejía D. A., Shelah S., *Preservation of splitting families and cardinal characteristics of the continuum*, Israel J. Math. **246** (2021), no. 1, 73–129.
- [13] Ihoda J. I., Shelah S., *Souslin forcing*, J. Symbolic Logic **53** (1988), no. 4, 1188–1207.
- [14] Jech T., *Set Theory*, Springer Monographs in Mathematics, Springer, Berlin, 2003.
- [15] Kunen K., Vaughan J. E., eds., *Handbook of Set-theoretic Topology*, North-Holland Publishing Co., Amsterdam, 1984.
- [16] Mejía D. A., *Matrix iterations and Cichon’s diagram*, Arch. Math. Logic **52** (2013), no. 3–4, 261–278.
- [17] Shelah S., *On cardinal invariants of the continuum*, Conf. Axiomatic Set Theory, Boulder, 1983, Contemp. Math., 31, Amer. Math. Soc., Providence, 1984, pages 183–207.