Douadi Drihem

Complex interpolation of function spaces with general weights

Comment.Math.Univ.Carolin. 64,3 (2023) 289–320.

Abstract: We present the complex interpolation of Besov and Triebel–Lizorkin spaces with generalized smoothness. In some particular cases these function spaces are just weighted Besov and Triebel–Lizorkin spaces. As a corollary of our results, we obtain the complex interpolation between the weighted Triebel–Lizorkin spaces $\dot{F}^{s_0}_{p_0,q_0}(\omega_0)$ and $\dot{F}^{s_1}_{\infty,q_1}(\omega_1)$ with suitable assumptions on the parameters s_0, s_1, p_0, q_0 and q_1, and the pair of weights (ω_0, ω_1).

Keywords: Besov space; Triebel–Lizorkin space; complex interpolation; Muckenhoupt class

AMS Subject Classification: 42B25, 42B35, 26B35, 46E35

References

[1] Andersen K. F., John R. T., *Weighted inequalities for vector-valued maximal functions and singular integrals*, Studia Math. **69** (1980/81), no. 1, 19–31.

[2] Bergh J., Löfström J., *Interpolation Spaces. An Introduction*, Grundlehren der Mathematischen Wissenschaften, 223, Springer, Berlin, 1976.

[3] Besov O. V., *Equivalent normings of spaces of functions of variable smoothness*, Funkts. Prostran., Priblizh., Differ. Uravn., Tr. Mat. Inst. Steklova **243** (2003), 87–95 (Russian); translation in Proc. Steklov Inst. Math. **243** (2003), no. 4, 80–88.

[4] Besov O. V., *Interpolation, embedding, and extension of spaces of functions of variable smoothness*, Issled. po Teor. Funkts. i Differ. Uravn., Tr. Mat. Inst. Steklova **248** (2005), 52–63 (Russian); translation in Proc. Steklov Inst. Math. **248** (2005), no. 1, 47–58.

[5] Bownik M., *Anisotropic Triebel–Lizorkin spaces with doubling measures*, J. Geom. Anal. **17** (2007), no. 3, 387–424.

[6] Bownik M., *Duality and interpolation of anisotropic Triebel–Lizorkin spaces*, Math. Z. **259** (2008), no. 1, 131–169.

[7] Bui H. Q., *Weighted Besov and Triebel spaces: interpolation by the real method*, Hiroshima Math. J. **12** (1982), no. 3, 581–605.

[8] Calderón A. P., *Intermediate spaces and interpolation, the complex method*, Studia Math. **24** (1964), 113–190.

[9] Cobos F., Fernandez D. L., *Hardy–Sobolev spaces and Besov spaces with a function parameter*, Function Spaces and Applications, Lund, 1986, Lecture Notes in Math., 1302, Springer, Berlin, 1988, pages 158–170.

[10] Domínguez O., Tikhonov S., *Function spaces of logarithmic smoothness: embeddings and characterizations*, Mem. Amer. Math. Soc. **282** (2023), no. 1393, vii+166 pages.

[11] Drihem D., *Besov spaces with general weights*, J. Math. Study. **56** (2023), no. 1, 18–92.

[12] Drihem D., *Triebel–Lizorkin spaces with general weights*, Adv. Oper. Theory **8** (2023) no. 1, Paper No. 5, 69 pages.

[13] Drihem D., *Duality of Triebel–Lizorkin spaces of general weights*, available at ArXiv: 2402.04635v1 [math.FA] (2024), 22 pages.

[14] Edmunds D., Triebel H., *Spectral theory for isotropic fractal drums*, C. R. Acad. Sci. Paris Sér. I Math. **326** (1998), no. 11, 1269–1274.

[15] Edmunds D., Triebel H., *Eigenfrequencies of isotropic fractal drums*, The Maz'ya anniversary collection, 2, Rostock, 1998, Birkhäuser Verlag, Basel, Oper. Theory Adv. Appl. **110** (1999), pages 81–102.

[16] Farkas W., Leopold H.-G., *Characterisations of function spaces of generalised smoothness*, Ann. Mat. Pura Appl. (4) **185** (2006), no. 1, 1–62.

[17] Fefferman C., Stein E. M., *Some maximal inequalities*, Amer. J. Math. **93** (1971), 107–115.

[18] Frazier M., Jawerth B., *Decomposition of Besov spaces*, Indiana Univ. Math. J. **34** (1985), no. 4, 777–799.

[19] Frazier M., Jawerth B., *A discrete transform and decomposition of distribution spaces*, J. Funct. Anal. **93** (1990), no. 1, 34–170.

[20] Frazier M., Jawerth B., Weiss G., *Littlewood–Paley Theory and the Study of Function Spaces*, CBMS Regional Conference Series in Mathematics, 79, Published for the Conference Board of the Mathematical Sciences, Washington, DC., American Mathematical Society, Providence, 1991.

[21] García-Cuerva J., Rubio de Francia J. L., *Weighted Norm Inequalities and Related Topics*, North-Holland Mathematics Studies, 116, Notas de Matemática, 104, North-Holland Publishing Co., Amsterdam, 1985.

[22] Goldman M. L., *Description of traces for certain function spaces*, Trudy Mat. Inst. Steklov. **150** (1979), 99–127, 322 (Russian).

[23] Goldman M. L., *The method of coverings for describing general spaces of Besov type*, Trudy Mat. Inst. Steklov. **156** (1980), 47–81, 262 (Russian).

[24] Grafakos L., *Classical Fourier Analysis*, Graduate Texts in Mathematics, 249, Springer, New York, 2014.

[25] Kaljabin G. A., *Descriptions of functions from classes of Besov–Lizorkin–Triebel type*, Trudy Mat. Inst. Steklov. **156** (1980), 82–109, 262 (Russian).

[26] Kaljabin G. A., Lizorkin P. I., *Spaces of functions of generalized smoothness*, Math. Nachr. **133** (1987), 7–32.

[27] Kalton N., Mayboroda S., Mitrea M., *Interpolation of Hardy–Sobolev–Besov–Triebel–Lizorkin spaces and applications to problems in partial differential equations*, Interpolation Theory and Applications, Amer. Math. Soc., Providence, Contemp. Math. **445** (2007), 121–177.

[28] Kempka H., Vybíral J., *Spaces of variable smoothness and integrability: characterizations by local means and ball means of differences*, J. Fourier Anal. Appl. **18** (2012), no. 4, 852–891.

[29] Kokilashvili V. M., *Maximum inequalities and multipliers in weighted Lizorkin–Triebel spaces*, Dokl. Akad. Nauk SSSR **239** (1978), no. 1, 42–45 (Russian).

[30] Moura S. D., *Function Spaces of Generalised Smoothness*, Dissertationes Math. (Rozprawy Mat.), 398, 2001, 88 pages.

[31] Muckenhoupt B., *Weighted norm inequalities for the Hardy maximal function*, Trans. Amer. Math. Soc. **165** (1972), 207–226.

[32] Rychkov V. S., *Littlewood–Paley theory and function spaces with A_p^{loc}-weights*, Math. Nachr. **224** (2001), no. 1, 145–180.

[33] Schott T., *Function spaces with exponential weights. II.*, Math. Nachr. **196** (1998), 231–250.

[34] Sickel W., Skrzypczak L., Vybíral J., *Complex interpolation of weighted Besov and Lizorkin–Triebel spaces*, Acta. Math. Sci. (Engl. Ser.) **30** (2014), no. 8, 1297–1323.

[35] Tang C., *A note on weighted Besov-type and Triebel–Lizorkin-type spaces*, J. Funct. Spaces Appl. **2013** (2013), Article ID 865835, 12 pages.

[36] Triebel H., *Interpolation Theory, Function Spaces, Differential Operators*, Deutscher Verlag der Wissenschaften, Berlin, 1978.

[37] Triebel H., *Theory of Function Spaces*, Monographs in Mathematics, 78, Birkhäuser Verlag, Basel, 1983.

[38] Triebel H., *Theory of Function Spaces. II.*, Monographs in Mathematics, 84, Birkhäuser Verlag, Basel, 1992.

[39] Tyulenev A. I, *Description of traces of functions in the Sobolev space with a Muckenhoupt weight*, Tr. Mat. Inst. Steklova **284** (2014), 288–303; translation in Proc. Steklov Inst. Math. **284** (2014), no. 1, 280–295.

[40] Tyulenev A. I., *Some new function spaces of variable smoothness*, Mat. Sb. **206** (2015), no. 6, 85–128; translation in Sb. Math. **206** (2015), no. 5–6, 849–891.

[41] Tyulenev A. I., *Besov-type spaces of variable smoothness on rough domains*, Nonlinear Anal. **145** (2016), 176–198.

[42] Tyulenev A. I., *On various approaches to Besov-type spaces of variable smoothness*, J. Math. Anal. Appl. **451** (2017), no. 1, 371–392.

[43] Wojciechowska A., *Multidimensional Wavelet Bases in Besov and Lizorkin–Triebel Spaces*, PhD. Thesis, Adam Mickiewicz University Poznań, Poznań, 2012.

[44] Yang D., Yuan W., Zhuo C., *Complex interpolation on Besov-type and Triebel–Lizorkin-type spaces*, Anal. Appl. (Singap). **11** (2013), no. 5, 1350021, 45 pages.