Gerd Herzog, Peer C. Kunstmann
Eventually positive elements in ordered Banach algebras

Abstract: In ordered Banach algebras, we introduce eventually and asymptotically positive elements. We give conditions for the following spectral properties: the spectral radius belongs to the spectrum (Perron–Frobenius property); the spectral radius is the only element in the peripheral spectrum; there are positive (approximate) eigenvectors for the spectral radius. Recently such types of results have been shown for operators on Banach lattices. Our results can be viewed as a complement, since our structural assumptions on the ordered Banach algebra are much weaker.

References

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