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Products of topological spaces and families of filters

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Abstract: We show that, under suitably general formulations, covering properties, accumulation properties and filter convergence are all equivalent notions. This general correspondence is exemplified in the study of products. We prove that a product is Lindelöf if and only if all subproducts by $\leq \omega_1$ factors are Lindelöf. Parallel results are obtained for final ω_n-compactness, $[\lambda, \mu]$-compactness, the Menger and the Rothberger properties.

Keywords: filter convergence; ultrafilter; product; subproduct; sequential compactness; sequencewise $\mathcal{P}$-compactness; Lindelöf property; final λ-compactness; $[\mu, \lambda]$-compactness; Menger property; Rothberger property
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