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A Kalmár-style completeness proof for the logics of the hierarchy $\mathbb{I}^n\mathbb{P}^k$

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Abstract: The logics of the family $\mathbb{I}^n\mathbb{P}^k := \{I^n P^k\}_{(n,k) \in \omega^2}$ are formally defined by means of finite matrices, as a simultaneous generalization of the weakly-intuitionistic logic I^1 and of the paraconsistent logic P^1 . It is proved that this family can be naturally ordered, and it is shown a sound and complete axiomatics for each logic of the form $I^n P^k$. The involved completeness proof showed here is obtained by means of a generalization of the well-known Kalmár’s method, usually applied for many-valued logics.

Keywords: mathematical logic; Kalmár’s completeness proof; many-valued logic

AMS Subject Classification: 03B50, 03B53

REFERENCES

- [1] Blok W. J., Pigozzi D., *Algebraizable logics*, Mem. Amer. Math. Soc. **77** (1989), no. 396, vi+78 pages.
- [2] Carnielli W. A., Coniglio M. E., Marcos J., *Logics of formal inconsistency*, in Handbook of Philosophical Logic, Springer, London, 2007, pages 1–93.
- [3] Carnielli W. A., Marcos J., *A taxonomy of C-systems*, in Paraconsistency: The Logical Way to the Inconsistent, Proc. of the 2nd World Congress on Paraconsistency, São Sebstião, 2000, Lecture Notes in Pure and Appl. Math., 228, Marcel Dekker, New York, 2002, pages 1–94.
- [4] Ciuciura J., *A lattice of the paracomplete calculi*, Log. Issled. **26** (2020), no. 1, 110–123.
- [5] Ciuciura J., *Sette’s calculus P^1 and some hierarchies of paraconsistent systems*, J. of Logic Comput. **30** (2020), no. 5, 1109–1124.
- [6] da Costa N. C. A., *On the theory of inconsistent formal systems*, Notre Dame J. Formal Logic **15** (1974), 497–510.
- [7] da Costa N. C. A., *Inconsistent Formal Systems*, Habilitation Thesis, Universidade Federal do Paraná, Curitiba, Brazil, 1993 (Portuguese).
- [8] Fernández V., *Society Semantics for n-valued Logics*, Master’s Thesis, UNICAMP, Brazil, 2001 (Portuguese).
- [9] Fernández V., *Fibring of Logics in the Leibniz Hierarchy*, Ph.D. Thesis, UNICAMP, Brazil, 2005 (Portuguese).
- [10] Fernández V. L., Coniglio M. E., *Combining valuations with society semantics*, Journal of Applied Non-Classical Logics **13** (2003), no. 1, 21–46.
- [11] Henkin L., *Fragments of the propositional calculus*, J. Symbolic Logic **14** (1949), 42–48.
- [12] L’abbé M., *On the independence of Henkin’s axioms for the fragments of the propositional calculus*, J. Symbolic Logic **16** (1951), 43–45.
- [13] Lewin R. A., Mikenberg I. F., Schwarze M. G., *Algebraization of paraconsistent logic P^1* , J. Non-Classical Logic **7** (1990), no. 1–2, 79–88.
- [14] Lewin R. A., Mikenberg I. F., Schwarze M. G., *P^1 algebras*, Studia Logica **53** (1994), no. 1, 21–28.
- [15] Lopes dos Santos L., *Constructive completeness proofs for positive propositional calculi*, in Proc. of the Third Brazilian Conf. on Mathematical Logic, Inst. Math., Fed. Univ. Pernambuco, Recife, 1979, (A. Arruda, N. da Costa, A. Sette eds.), Soc. Brasil. Lógica, São Paulo, 1980, pages 199–209.
- [16] Mendelson E., *Introduction to Mathematical Logic*, Chapman and Hall, London, 1997.
- [17] Olvera Badillo A., *Revisiting Kalmár completeness metaproof*, Proc. of the 6th Latin American Workshop on Logic/Languages, Algorithms and New Methods of Reasoning Puebla, Mexico, 2010, vol. 677 of CEUR Workshop Proceedings, 2010, pages 99–106.
- [18] Pynko A. P., *Algebraic study of Sette’s maximal paraconsistent logic*, Studia Logica **54** (1995), no. 1, 89–128.
- [19] Ramos F. M., Fernández V. L., *Twist-structures semantics for the logics of the hierarchy $I^n P^k$* , J. Appl. Non-Classical Logics **19** (2009), no. 2, 183–209.
- [20] Sette A. M., *On the propositional calculus P^1* , Math. Japon. **18** (1973), 173–180.

- [21] Sette A.-M., Carnielli W. A., *Maximal weakly-intuitionistic logics*, *Studia Logica* **55** (1995), no. 1, 181–203.
- [22] Wójcicki R., *Theory of Logical Calculi*, Basic Theory of Consequence Operations, Synthese Library, 199, Kluwer Academic Publishers Group, Dordrecht, 1988.