

## Hichem Khelifi, Youssef El Hadfi

### *New nonlinear Picone identities with variable exponents and applications*

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**Abstract:** This paper introduces two novel nonlinear anisotropic Picone identities with variable exponents that expand upon the traditional identity used for the ordinary Laplace equation. Additionally, the research explores potential applications of these findings in anisotropic Sobolev spaces featuring variable exponents.

**Keywords:** anisotropic Picone identity; variable exponent

**AMS Subject Classification:** 35A23, 35A02, 35J75

#### REFERENCES

- [1] Abramowitz M., Stegun I. A., *Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables*, National Bureau of Standards Applied Mathematics Series, 55, U.S. Government Printing Office, Washington, 1964.
- [2] Allegretto W., *Form estimates for the  $p(x)$ -Laplacean*, Proc. Amer. Math. Soc. **135** (2007), no. 7, 2177–2185.
- [3] Allegretto W., Huang Y. X., *A Picone's identity for the  $p$ -Laplacian and applications*, Nonlinear Anal. **32** (1998), no. 7, 819–830.
- [4] Boccardo L., Gallouët T., Marcellini P., *Anisotropic equations in  $L^1$* , Differential Integral Equations **9** (1996), no. 1, 209–212.
- [5] Chen Y., Levine S., Rao M., *Variable exponent, linear growth functionals in image restoration*, SIAM J. Appl. Math. **66** (2006), no. 4, 1383–1406.
- [6] Fan X., *Anisotropic variable exponent Sobolev spaces and  $p_i(x)$ -Laplacian equations*, Complex Var. Elliptic Equ. **56** (2011), no. 7–9, 623–642.
- [7] Feng T., Cui X., *Anisotropic Picone identities and anisotropic Hardy inequalities*, J. Inequal. Appl. **2017** (2017), Paper No. 16, 9 pages.
- [8] Feng T., Han J., *A new variable exponent Picone identity and applications*, Math. Inequal. Appl. **22** (2019), no. 1, 65–75.
- [9] Feng T., Zhang K., *A nonlinear Picone identity for anisotropic Laplace operator and its applications*, J. of Math. (PRC). **40** (2020), no. 3, 283–290.
- [10] Jikov V. V., Kozlov S. M., Oleĭnik O. A., *Homogenization of Differential Operators and Integral Functionals*, Springer, Berlin, 1994.
- [11] Khelifi H., *Anisotropic degenerate elliptic problem with singular gradient lower order term*, Boll. Unione Mat. Ital. **17** (2024), no. 1, 149–174.
- [12] Khelifi H., *Anisotropic parabolic-elliptic systems with degenerate thermal conductivity*, accepted at Applicable Analysis (2023), 33 pages.
- [13] Khelifi H., Ait-Mahiout K., *Regularity for solutions of elliptic  $p(x)$ -Laplacian type equations with lower order terms and Hardy potential*, accepted at Ricerche Mat. (2023).
- [14] Khelifi H., El Hadfi Y., *Nonlinear elliptic equations with variable exponents involving singular nonlinearity*, Math. Model. Comput. **8** (2021), no. 4, 705–715.
- [15] Khelifi H., Mokhtari F., *Nonlinear degenerate anisotropic elliptic equations with variable exponents and  $L^1$  data*, J. Part. Diff. Eq. **33** (2020), no. 1, 1–16.
- [16] Kováčik O., Rákosník J., *On spaces  $L^{p(x)}$  and  $W^{k,p(x)}$* , Czechoslovak Math. J. **41(116)** (1991), no. 4, 592–618.
- [17] Mihăilescu M., Pucci P., Rădulescu V., *Nonhomogeneous boundary value problems in anisotropic Sobolev spaces*, C. R. Math. Acad. Sci. Paris **345** (2007), no. 10, 561–566.
- [18] Mihăilescu M., Rădulescu V., *On a nonhomogeneous quasilinear eigenvalue problem in Sobolev spaces with variable exponent*, Proc. Amer. Math. Soc. **135** (2007), no. 9, 2929–2937.
- [19] Naceri M., *Singular anisotropic elliptic problems with variable exponents*, Mem. Differ. Equ. Math. Phys. **85** (2022), 119–132.

- [20] Picone M., *Sui valori eccezionali di un parametro da cui dipende un'equazione differenziale lineare ordinaria del secondo ordine*, Ann. Scuola Norm. Sup. Pisa Cl. Sci. **11** (1910), 144 pages (Italian).
- [21] Picone M., *Un teorema sulle soluzioni delle equazioni lineari ellittiche autoaggiunte alle derivate parziali del secondo ordine*, Rend. Mat. Acc. Lincei **17** (1911), no. 5, 213–219 (Italian).
- [22] Yoshida N., *Picone identity for quasilinear elliptic equations with  $p(x)$ -Laplacians and Sturmian comparison theory*, Appl. Math. Comput. **225** (2013), 79–91.
- [23] Zouatini M.A., Mokhtari F., Khelifi H., *Degenerate elliptic problem with singular gradient lower order term and variable exponents*, Math. Model. Comput. **10** (2023), no. 1, 133–146.