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The positive cone of a Banach lattice. Coincidence of topologies and metrizability

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Abstract: Let X be a Banach lattice, and denote by X_+ its positive cone. The weak topology on X_+ is metrizable if and only if it coincides with the strong topology if and only if X is Banach-lattice isomorphic to $l^1(\Gamma)$ for a set Γ. The weak* topology on X_+^* is metrizable if and only if X is Banach-lattice isomorphic to a $C(K)$-space, where K is a metrizable compact space.

Keywords: normed lattice; Banach lattice; positive cone; AM-space; AL-space; Banach lattice $C(K)$; Banach lattice $l^1(\Gamma)$; strong topology; weak topology; weak* topology; coincidence of topologies; metrizability; nonatomic measure
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