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On a probabilistic problem on finite semigroups

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Abstract: We deal with the following problem: how does the structure of a finite semigroup S depend on the probability that two elements selected at random from S , with replacement, define the same inner right translation of S . We solve a subcase of this problem. As the main result of the paper, we show how to construct not necessarily finite medial semigroups in which the index of the kernel of the right regular representation equals two.

Keywords: semigroup; regular representation of semigroups; medial semigroup

AMS Subject Classification: 20M10, 20M15

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