Rachid Tribak, Yahya Talebi, Mehrab Hosseinpour
Some results on quasi-t-dual Baer modules

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Abstract: Let R be a ring and let M be an R-module with $S = \mathrm{End_R(M)}$. Consider the preradical $\overline{Z}$ for the category of right R-modules Mod-R introduced by Y. Talebi and N. Vanaja in 2002 and defined by $\overline{Z}(M) = \bigcap \{U \leq M \colon M/U \text{ is small in its injective hull}\}$. The module M is called quasi-t-dual Baer if $\sum_{\varphi \in \mathfrak{I}} \varphi(\overline{Z}^2(M))$ is a direct summand of M for every two-sided ideal $\mathfrak{I}$ of S, where $\overline{Z}^2(M) = \overline{Z}(\overline{Z}(M))$. In this paper, we show that M is quasi-t-dual Baer if and only if $\overline{Z}^2(M)$ is a direct summand of M and $\overline{Z}^2(M)$ is a quasi-dual Baer module. It is also shown that any direct summand of a quasi-t-dual Baer module inherits the property. The last part of the paper is devoted to the comparison of the notions of quasi-dual Baer modules and quasi-t-dual Baer modules. Also, right quasi-t-dual Baer rings are investigated.

Keywords: fully invariant submodule; quasi-dual Baer module; quasi-dual Baer ring; quasi-t-dual Baer module; quasi-t-dual Baer ring
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