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About wcs-covers and wcs*-networks on the Vietoris hyperspace $\mathcal{F}(X)$

Abstract: We study some generalized metric properties on the hyperspace $\mathcal{F}(X)$ of finite subsets of a space X endowed with the Vietoris topology. We prove that X has a point-star network consisting of (countable) *wcs*-covers if and only if so does $\mathcal{F}(X)$. Moreover, X has a sequence of *wcs*-covers with property (P) which is a point-star network if and only if so does $\mathcal{F}(X)$, where (P) is one of the following properties: point-finite, point-countable, compact-finite, compact-countable, locally finite, locally countable. On the other hand, X has a *wcs**-network with property σ-(P) if and only if so does $\mathcal{F}(X)$. By these results, we obtain some results related to the images of metric spaces and separable metric spaces under some kinds of continuous mappings on the Vietoris hyperspace $\mathcal{F}(X)$.

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