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## *Inequalities involving norm and numerical radius of Hilbert space operators*

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**Abstract:** This paper presents several numerical radii and norm inequalities for Hilbert space operators. These inequalities improve some earlier related inequalities. For an operator  $A$ , we prove that

$$\omega^2(A) \leq \left\| \frac{A^*A + AA^*}{2} - \frac{1}{2R} \left( (1-t)A^*A + tAA^* - ((1-t)(A^*A)^{1/2} + (AA^*)^{1/2})^2 \right) \right\|$$

where  $R = \max\{t, 1-t\}$  and  $0 \leq t \leq 1$ .

**Keywords:** bounded linear operator; numerical radius; operator norm; inequality

**AMS Subject Classification:** 47A12, 47A30, 47A63

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